

## **GPS Satellite Clock Estimation Every 30 Seconds and Application to Accurate Absolute Positioning**

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**August 1998**

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**20020828 039**



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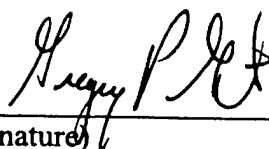
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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE April 1999	3. REPORT TYPE AND DATES COVERED Scientific, Final, 12 Sep 1995-5 Jul 1998		
4. TITLE AND SUBTITLE GPS Satellite Clock Estimation Every 30 Seconds and Application to Accurate Absolute Positioning		5. FUNDING NUMBERS Contract F19628-96-C-0169 PE 663200 PR DMAP TA GL WU AB		
6. AUTHOR(S) Shin-Chan Han Christopher Jekeli				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Ohio State University Research Foundation 1960 Kenny Rd Columbus, OH 43210-1063		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  Air Force Research Laboratory 29 Randolph Rd Hanscom AFB, MA 01731-3010 Contract Manager: Anestis Romaides/VSBXI		10. SPONSORING/MONITORING AGENCY REPORT NUMBER  AFRL-VS-TR-2000-1607		
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION AVAILABILITY STATEMENT Approved for Public Release; distribution unlimited			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This report presents the results of investigations to determine very accurate position coordinates using the Global Positioning System in the absolute (point) positioning mode. The most common method to obtain very accurate positions with GPS is to apply double-differencing procedures whereby GPS satellite signals are differenced at a station and these differences are again differenced with analogous differences at other stations. The differencing between satellites eliminates the large receiver clock errors, while the between-station differences eliminate the large satellite clock errors (as well as some other errors, such as orbit error). However, only coordinate differences can be determined in this way and the accuracy depends on the baseline length between cooperating stations. The strategy with accurate point positioning is to estimate GPS satellite clock errors independently, thus obviating the between-station differencing. The clock error estimates are then used in an application of a single-difference (between-satellite) positioning algorithm at any site to determine the coordinates without reference to any other site. Using IGS orbits and stations, the GPS clock errors were estimated at 30-second intervals and these estimates were compared to values determined by JPL. The agreement was at the level of about 0.1 nsec (3 cm). The absolute positioning technique was tested at a stationary site (IGS station) whose coordinates are known. The differences between the estimated absolute position coordinates and the known values had a standard deviation less than 4 cm in all three dimensions, with mean differences ranging from 3.4 cm to 6.3 cm.				
14. SUBJECT TERMS  Global Positioning System   Error estimates   Absolute (point) positioning			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT  UNCL	18. SECURITY CLASSIFICATION OF THIS PAGE  UNCL	19. SECURITY CLASSIFICATION OF ABSTRACT  UNCL	20. LIMITATION OF ABSTRACT  UNL	

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## **Abstract**

This report presents the results of investigations to determine very accurate position coordinates using the Global Positioning System in the absolute (point) positioning mode. The most common method to obtain very accurate positions with GPS is to apply double-differencing procedures whereby GPS satellite signals are differenced at a station and these differences are again differenced with analogous differences at other stations. The differencing between satellites eliminates the large receiver clock errors, while the between-station differences eliminate the large satellite clock errors (as well as some other errors, such as orbit error). However, only coordinate differences can be determined in this way and the accuracy depends on the baseline length between cooperating stations. The strategy with accurate point positioning is to estimate GPS satellite clock errors independently, thus obviating the between-station differencing. The clock error estimates are then used in an application of a single-difference (between-satellite) positioning algorithm at any site to determine the coordinates without reference to any other site. Using IGS orbits and stations, the GPS clock errors were estimated at 30-second intervals and these estimates were compared to values determined by JPL. The agreement was at the level of about 0.1 nsec (3 cm). The absolute positioning technique was tested at a stationary site (IGS station) whose coordinates are known. The differences between the estimated absolute position coordinates and the known values had a standard deviation less than 4 cm in all three dimensions, with mean differences ranging from 3.4 cm to 6.3 cm.

## **1. Introduction**

One of the important unknowns in the Global Positioning System (GPS) is the satellite clock error (the variations of the nominal time with respect to GPS time). Part of this is due to random errors in the clocks and partly it is an intentional dithering, called Selective Availability (SA), that degrades the signal accuracy up to 60 meters. Geodesists have circumvented this error by using differential techniques whereby the signals from satellites at two stations are differenced, thus eliminating the common satellite and receiver clock errors. In addition, GPS orbit errors also tend to cancel. However, this relative positioning of one station with respect to the other has its limitations and the accuracies depend strongly on the baseline length between stations. One of the more critical aspects is the correct modeling of the tropospheric effect at the two stations, where errors in the model do not cancel if the stations are far apart (several 100 km).

If the satellite clock errors and the orbits were determined with certain accuracies, one could do absolute GPS positioning (using only one receiver) in either the static or the dynamic mode. This would have tremendous importance in many applications in geodesy and other disciplines that require accurate positioning in remote areas. The quality of the orbits and clock errors calculated from the navigation message, however, are not accurate enough for precise absolute positioning

because of SA (intentional dithering of the fundamental frequency of GPS and the manipulation of the ephemeris). Therefore, it is necessary to estimate the GPS orbits and the clock errors in order to conduct accurate absolute positioning.

Nowadays, the International GPS Service (IGS) analysis center provides GPS satellite orbit and clock error estimations at 900-second intervals with about 5 and 10 cm accuracy, respectively (IGS, 1999). It should be noted that orbits and clock errors at a much higher rate (up to 1 Hz) are required in some applications related to accurate positioning of moving-base platforms such as aerial photogrammetry, sea-level monitoring using ocean buoys, airborne vector gravimetry, and other remote sensing systems.

GPS ephemerides at higher rates, such as 30 sec (0.033 Hz), equal to the data rate of the current IGS stations' observation, can be obtained by an interpolation of the 900-sec ephemerides within centimeter level accuracy (Remondi, 1989, 1991). However, the interpolation is not feasible to obtain the satellite clock errors because the SA effects on the clock errors have significant variations at periods down to 2 min. In this paper, a method is introduced (for application to post-processing and interpolated orbits) to estimate the satellite clock error every 30 sec using the observations from the globally distributed IGS control stations (IGS, 1999). Once the satellite clock error is determined, it can be used in precise absolute positioning.

## 2. Mathematical Derivation

### 2.1. The Observation Equations

The observation equations of the four GPS measurement types are given as follows (Goad, 1995):

$$\Phi_{i,1}^k(t) = \rho_i^k(t) + c \left( dt_i(t) - dt^k(t) \right) + T_i^k(t) - \frac{I_i^k(t)}{f_1^2} + \lambda_1 N_{i,1}^k + \lambda_1 \left( \phi_i(t_0) - \phi^k(t_0) \right)_1 + \epsilon_{i,1}^k \quad (1)$$

$$\Phi_{i,2}^k(t) = \rho_i^k(t) + c \left( dt_i(t) - dt^k(t) \right) + T_i^k(t) - \frac{I_i^k(t)}{f_2^2} + \lambda_2 N_{i,2}^k + \lambda_2 \left( \phi_i(t_0) - \phi^k(t_0) \right)_2 + b_{i,1}^k(t) + \epsilon_{i,2}^k \quad (2)$$

$$P_{i,1}^k(t) = \rho_i^k(t) + c \left( dt_i(t) - dt^k(t) \right) + T_i^k(t) + \frac{I_i^k(t)}{f_1^2} + b_{i,2}^k(t) + \epsilon_{i,1}^k \quad (3)$$

$$P_{i,2}^k(t) = \rho_i^k(t) + c \left( dt_i(t) - dt^k(t) \right) + T_i^k(t) + \frac{I_i^k(t)}{f_2^2} + b_{i,3}^k(t) + \epsilon_{i,2}^k \quad (4)$$

Here, the subscript  $i$  indicates the index for the receiver and superscript  $k$  does that for the satellite.

$c$  is the speed of light;  $f_1$  and  $f_2$  are the L1 and L2 carrier frequencies;  $\lambda_1$  and  $\lambda_2$  are the L1 and L2 carrier wavelengths;  $\Phi_{i,1}^k$ ,  $\Phi_{i,2}^k$ ,  $P_{i,1}^k$ , and  $P_{i,2}^k$  are the phase range and pseudorange measurements from the satellite  $k$  and at the receiver  $i$ ;  $\rho_i^k$  is the geometric distance between the satellite's antenna at the signal transmission time and the receiver's antenna position at the signal reception time;  $T_i^k$  is the tropospheric delay;  $I_i^k$  is the frequency-dependent ionospheric refraction and  $N_{i,1}^k$  and  $N_{i,2}^k$  are the ambiguities of the L1 and L2 phase measurements. The one-way phase observables are additionally involved with a fixed nonzero initial fractional phase term  $\lambda(\phi_i(t_0) - \phi^k(t_0))$  that is contained in the receiver- and satellite-generated phase signals. The remaining terms  $b_{i,1}^k$ ,  $b_{i,2}^k$ , and  $b_{i,3}^k$  are the relative interchannel biases between  $\Phi_{i,1}^k$  and  $\Phi_{i,2}^k$ ,  $P_{i,1}^k$ , and  $P_{i,2}^k$ , respectively. They result from the fact that the L1 and L2 signals travel through different hardware paths inside the receiver as well as the satellite transmitter (Coco, 1991). Therefore, the interchannel biases are dependent on both the satellite and the receiver.

While the magnitude of the phase observation noise is about a millimeter, that of the P and C/A code noises are much larger depending on the type of receiver. Generally, P-code noise is about 30 cm, and C/A code noise can be a meter or more. From the above four measurement equations, some combinations are possible for eliminating the nuisance parameters such as the ionospheric refraction, the receiver clock errors, and the ambiguities.

## 2.2. Ion-free, Wide-Lane Combination

As mentioned above, the ionospheric effect depends on the frequency of the signal. Thus, by using dual frequency signals, it is possible to eliminate the first order ionospheric effect by a combination of phase and code measurements. Because the maximum contributions of the 2nd and 3rd order terms of this effect are about 3 cm and less than 1 cm, respectively (Seeber, 1993), eliminating the first order effect might be enough for most applications. The so-called ion-free, wide-lane signal (86 cm wavelength) can be obtained by first multiplying the combination coefficients  $f_1^2/(c(f_1 + f_2)) \approx 2.95$  and  $f_2^2/(c(f_1 + f_2)) \approx 1.79$  to equation (1) and (2) or (3) and (4), and then taking the differences between L1 and L2 measurements. The wide-lane combination is known to be less sensitive to the noises because of its longer wavelength (Hofmann-Wellenhof et al., 1992). We have for the carrier phase:

$$\begin{aligned}\Phi_{i,\text{ion-free}}^k(t) &= \frac{f_1}{f_1 + f_2} \frac{f_1}{c} \Phi_{i,1}^k(t) - \frac{f_2}{f_1 + f_2} \frac{f_2}{c} \Phi_{i,2}^k(t) \\ &= \frac{f_1 - f_2}{c} \rho_i^{*k}(t) + (f_1 - f_2) (dt_i(t) - dt^k(t)) + \frac{f_1}{f_1 + f_2} N_{i,1}^{*k} - \frac{f_2}{f_1 + f_2} N_{i,2}^{*k} + b_{i,\text{phase}}^k(t) + \epsilon_i^k\end{aligned}\quad (5)$$

and for the pseudo-range phase:

$$\begin{aligned}
R_{i,\text{ion-free}}^k(t) &= \frac{f_1}{f_1 + f_2} \frac{f_1}{c} P_{i,1}^k(t) - \frac{f_2}{f_1 + f_2} \frac{f_2}{c} P_{i,2}^k(t) \\
&= \frac{f_1 - f_2}{c} \rho_i^{*k}(t) + (f_1 - f_2) (dt_i(t) - dt^k(t)) + b_{i,\text{code}}^k(t) + e_i^k
\end{aligned} \tag{6}$$

Note that  $\rho_i^{*k}$  includes the geometric range  $\rho_i^k$  and tropospheric delay  $T_i^k$ . Furthermore,  $N_i^{*k}$  is no longer an integer and consists of the integer ambiguity  $N_i^k$  and the fractional phase offset  $\lambda(\phi_i(t_0) - \phi^k(t_0))$ . The interchannel bias is scaled by the combination of coefficients and the magnitude of the noise is decreased by the factor of 0.7.

### 2.3. The Satellite Differenced Measurement

For one receiver tracking two satellites ( $k$ th and  $l$ th) simultaneously, satellite-single-differenced measurements are obtained. This single differencing eliminates the receiver dependent effects such as the receiver clock error  $dt_i$ , the non-zero initial phase offset of the receiver  $\phi_i(t_0)$ , and the interchannel biases of the receiver  $b_i$ . By taking the difference of ion-free, wide-lane combinations between the  $k$ th and  $l$ th satellites, we obtain:

$$\begin{aligned}
\phi_{i,\text{ion-free}}^{k,l}(t) &= \phi_{i,\text{ion-free}}^k(t) - \phi_{i,\text{ion-free}}^l(t) \\
&= \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t) - (f_1 - f_2) dt^{k,l}(t) + N_w^{*k,l} + b_{\text{phase}}^{k,l}(t) + \epsilon
\end{aligned} \tag{7}$$

$$\begin{aligned}
R_{i,\text{ion-free}}^{k,l}(t) &= R_{i,\text{ion-free}}^k(t) - R_{i,\text{ion-free}}^l(t) \\
&= \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t) - (f_1 - f_2) dt^{k,l}(t) + b_{\text{code}}^{k,l}(t) + e
\end{aligned} \tag{8}$$

$$\text{where } N_w^{*k,l} = \left( \frac{f_1}{f_1 + f_2} N_{i,1}^{*k,l} - \frac{f_2}{f_1 + f_2} N_{i,2}^{*k,l} \right).$$

The remaining unknowns are the satellite clock error, two satellite dependent interchannel biases, and the wide-lane ambiguity. One thing should be noted is that the interchannel biases can not be separated from the satellite clock error and the ambiguity.

### 2.4. The Time Differenced Measurements

Assuming the measurements have no cycle slips, the ambiguity will remain constant and it can be eliminated when two independent measurements are differenced with respect to time. Similarly,



the interchannel bias term could be eliminated by differencing with respect to time if we assume it is constant. Suppose that phase measurements are obtained for two consecutive epochs ( $t_i$  and  $t_j$ ) without cycle slip. Then

$$\begin{aligned}\phi_{i,\text{ion-free}}^{k,\ell}(t_{i,j}) &= \phi_{i,\text{ion-free}}^k(t_i) - \phi_{i,\text{ion-free}}^\ell(t_j) \\ &= \frac{f_1 - f_2}{c} \rho_i^{*,k,\ell}(t_{i,j}) - (f_1 - f_2) dt^{k,\ell}(t_{i,j}) + b_{\text{phase}}^{k,\ell}(t_i) - b_{\text{phase}}^{k,\ell}(t_j) + \varepsilon\end{aligned}\quad (9)$$

where  $\rho_i^{*,k,\ell}(t_{i,j}) = \rho_i^{*,k,\ell}(t_i) - \rho_i^{*,k,\ell}(t_j)$  and  $dt^{k,\ell}(t_{i,j}) = dt^{k,\ell}(t_i) - dt^{k,\ell}(t_j)$ .

Here, the difference between the two consecutive satellite interchannel biases is assumed to be a stochastic process with zero mean. It is reasonable to neglect this difference, since its behavior is known to be quite stable from one day to the next (Joachim Feltens at ESOC, personal communication, 1999; Wilson and Mannucci, 1993). Therefore, we have

$$\phi_{i,\text{ion-free}}^{k,\ell}(t_{i,j}) = \frac{f_1 - f_2}{c} \rho_i^{*,k,\ell}(t_{i,j}) - (f_1 - f_2) dt^{k,\ell}(t_{i,j}) + \varepsilon \quad (10)$$

Now, in the above equation, two nuisance parameters, namely the receiver clock error and the ambiguity do not exist any more. Using IGS globally distributed stations coordinates,  $\rho_i^{*,k,\ell}$  can be computed (using a suitable tropospheric model at each station), and we have the measurement  $\phi_{i,\text{ion-free}}^{k,\ell}$ . Thus, the only unknown parameter is the single differenced GPS clock error  $dt^{k,\ell}$ . Rearranging equation (10) in terms of the unknown quantity  $dt^{k,\ell}$ , yields:

$$dt^{k,\ell}(t_{i,j}) = \frac{1}{c} \rho_i^{*,k,\ell}(t_{i,j}) - \frac{1}{f_1 - f_2} \phi_{i,\text{ion-free}}^{k,\ell}(t_{i,j}) + \varepsilon \quad (11)$$

## 2.5. The Satellite Clock Error and Absolute Positioning

The satellite-differenced, time-differenced, ion-free, phase combination produces the relative variations of the single-differenced satellite clock error with respect to the initial epoch. Suppose that the phase measurements are obtained at an IGS control station for  $n$  epochs. For epoch  $t_1$  and  $t_2$ , the time-differenced clock error,  $dt^{k,\ell}(t_{2,1})$ , is estimated from the equation (11). Then the clock error for the next epoch  $t_2$ ,  $dt^{k,\ell}(t_2)$ , can be expressed in terms of the initial clock error,  $dt^{k,\ell}(t_1)$ , as follows:

$$dt^{k,\ell}(t_2) = dt^{k,\ell}(t_1) + dt^{k,\ell}(t_{2,1}) \quad (12)$$

For epoch  $t_2$  and  $t_3$ ,

$$dt^{k,l}(t_3) = dt^{k,l}(t_2) + dt^{k,l}(t_{3,2}) = dt^{k,l}(t_1) + dt^{k,l}(t_{2,1}) + dt^{k,l}(t_{3,2}) \quad (13)$$

In general,

$$dt^{k,l}(t_n) = dt^{k,l}(t_1) + \sum_{m=2}^n dt^{k,l}(t_{m,m-1}) \quad (14)$$

Therefore, if the satellite clock error at an initial or an arbitrary epoch is available, the satellite clock errors for all epochs are calculated according to equation (14). However, there is no need to know the initial satellite clock error in absolute positioning as described in the next paragraph. It is reasonable because the initial clock error can be absorbed into the ambiguity term in the absolute positioning procedure.

Assume that the relative satellite clock error was estimated and measurements of phase were obtained at the unknown sites, whose coordinates are to be determined. After taking the ion-free, wide-lane combination and calculating single differences between satellites, the measurements are described as follows:

$$\phi_{i,\text{ion-free}}^{k,l}(t_1) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_1) - (f_1 - f_2) dt^{k,l}(t_1) + N_w^{*k,l} + b_{\text{phase}}^{k,l}(t_1) + \varepsilon \quad (15)$$

$$\phi_{i,\text{ion-free}}^{k,l}(t_2) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_2) - (f_1 - f_2) dt^{k,l}(t_2) + N_w^{*k,l} + b_{\text{phase}}^{k,l}(t_2) + \varepsilon \quad (16)$$

...

$$\phi_{i,\text{ion-free}}^{k,l}(t_n) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_n) - (f_1 - f_2) dt^{k,l}(t_n) + N_w^{*k,l} + b_{\text{phase}}^{k,l}(t_n) + \varepsilon \quad (17)$$

By putting the estimated satellite clock errors  $dt^{k,l}(t_m)$  into equations (15), (16) and (17), we find:

$$\phi_{i,\text{ion-free}}^{k,l}(t_1) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_1) - (f_1 - f_2) dt^{k,l}(t_1) + N_w^{*k,l} + b_{\text{phase}}^{k,l} + \varepsilon \quad (18)$$

$$\phi_{i,\text{ion-free}}^{k,l}(t_2) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_2) - (f_1 - f_2) dt^{k,l}(t_{2,1}) - (f_1 - f_2) dt^{k,l}(t_1) + N_w^{*k,l} + b_{\text{phase}}^{k,l} + \varepsilon \quad (19)$$

...

$$\phi_{i,\text{ion-free}}^{k,l}(t_n) = \frac{f_1 - f_2}{c} \rho_i^{*k,l}(t_n) - (f_1 - f_2) \sum_{m=2}^n dt^{k,l}(t_{m,m-1}) - (f_1 - f_2) dt^{k,l}(t_1) + N_w^{*k,l} + b_{\text{phase}}^{k,l} + \varepsilon \quad (20)$$

Now, one can define the new time constant variable,  $\tilde{N}_w^{*k,l}$ , which includes the ambiguity, the satellite interchannel bias (assumed constant), and the initial satellite clock error:

$$\tilde{N}_w^{*k,l} = N_w^{*k,l} + b_{\text{phase}}^{k,l} - (f_1 - f_2) dt^{k,l}(t_1) \quad (21)$$

By using this time independent variable (assuming no cycle slips and constant interchannel bias), equations (18), (19), and (20) are represented as follows:

$$\phi_{i,\text{ion-free}}^{k,l}(t_1) - \frac{f_1 - f_2}{c} T_i^{k,l}(t_1) = \frac{f_1 - f_2}{c} \rho_i^{k,l}(t_1) + \tilde{N}_w^{*k,l} + \varepsilon \quad (22)$$

$$\phi_{i,\text{ion-free}}^{k,l}(t_2) + (f_1 - f_2) dt^{k,l}(t_{2,1}) - \frac{f_1 - f_2}{c} T_i^{k,l}(t_2) = \frac{f_1 - f_2}{c} \rho_i^{k,l}(t_2) + \tilde{N}_w^{*k,l} + \varepsilon \quad (23)$$

...

$$\phi_{i,\text{ion-free}}^{k,l}(t_n) + (f_1 - f_2) \sum_{m=2}^n dt^{k,l}(t_{m,m-1}) - \frac{f_1 - f_2}{c} T_i^{k,l}(t_n) = \frac{f_1 - f_2}{c} \rho_i^{k,l}(t_n) + \tilde{N}_w^{*k,l} + \varepsilon \quad (24)$$

In the above equations, the unknowns are the newly defined ambiguity term and the position of the moving receiver  $x_i(t)$ ,  $y_i(t)$ , and  $z_i(t)$ , as contained in the range,  $\rho_i^{k,l}(t)$ . With the measurement  $\phi_{i,\text{ion-free}}^{k,l}(t)$ , estimated clock error  $dt^{k,l}(t_{m,m-1})$  and modeled tropospheric effect  $T_i^{k,l}(t)$ , those unknowns can be determined.

### 3. Procedure

The overall procedure of this method is depicted in Figure 1. The first step in this algorithm is to calculate the correction derived from a periodic relativistic effect (see Appendix) using the eccentricity, the semi-major axis, and the eccentric anomaly of the GPS satellite orbits, which are transmitted in the navigation message every 2 hours.

Next, the orbit of the GPS satellite at signal emission time is calculated by using the IGS precise orbit; and then, the time differencing on all station observations is performed, where the coordinates of the stations are known. The products of this process are the time-differenced GPS satellite clock errors including the receiver clock error (the receiver clock error is not eliminated in this step).

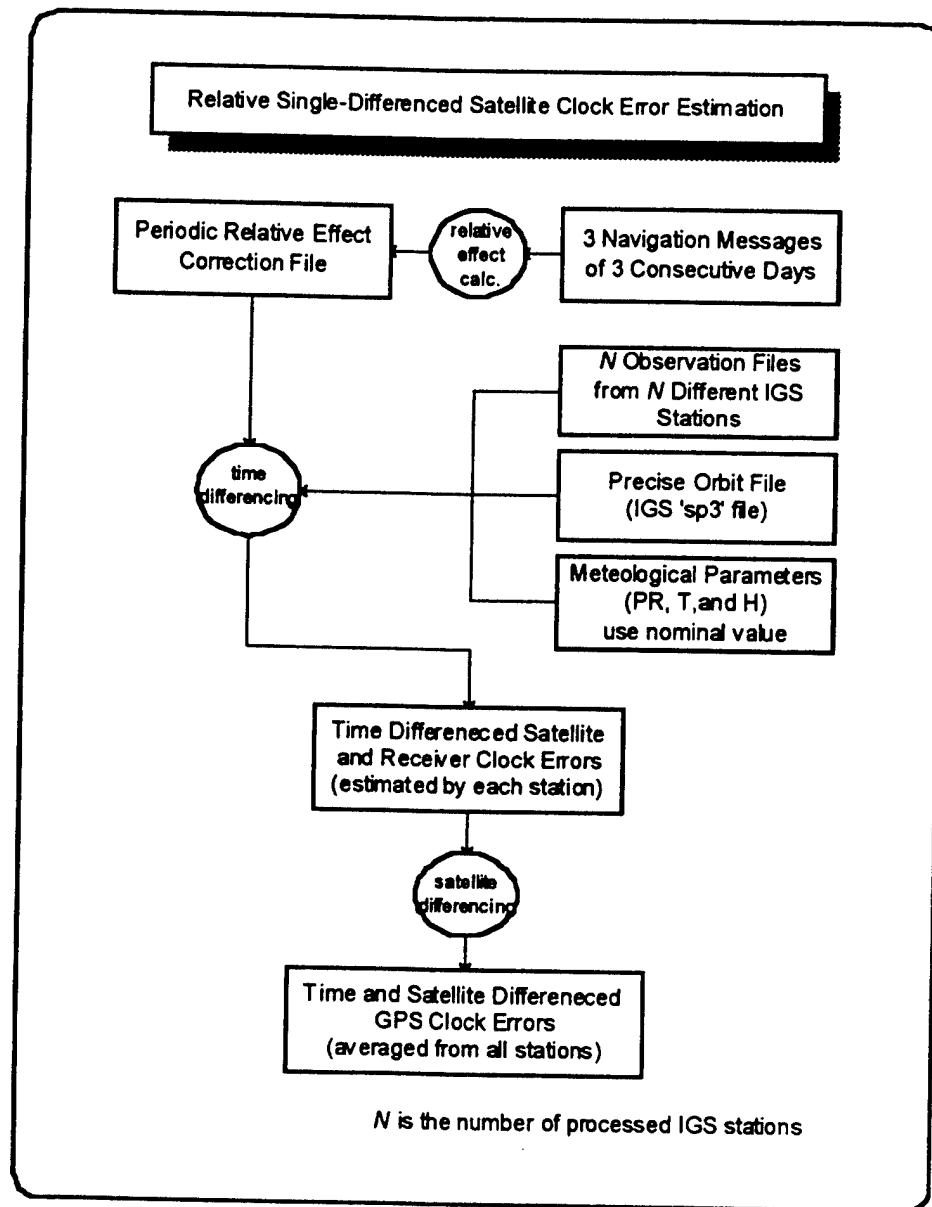


Figure 1

For the orbit calculation, the orbit's smooth behavior makes the interpolation possible within a certain accuracy. According to the studies by Remondi (1989, 1991), a 9th-order interpolator is sufficient for an accuracy of about 10 cm and with a 17th-order interpolator he demonstrates that millimeter-level accuracy can be achieved based on a 40-minute epoch interval. For the tropospheric delay, the modified Hopfield model (Goad and Goodman, 1974) is used.

Finally, after the satellite differencing, the receiver clock error is eliminated by differencing between satellites. The product of this process is the time- and satellite-differenced GPS satellite clock error. The redundant data (the same differences of clock errors can be obtained from other stations) for this estimation are sequentially averaged with equal weights. This assumes that the

clock errors determined using different stations are independent; however, they are not completely uncorrelated since there are slight differences in transmission time for the signal received by stations at the same time. That is, a station on the equator and a station on the pole will receive the GPS signal simultaneously transmitted at slightly different emission times (0.02 seconds of difference). It is assumed that the satellite clocks do not show such a high frequency variation for such a short time period. Because of this assumption, one also can estimate the satellite clock error at the receiving time by using the signals having the information about the satellite clock error at the emission time.

#### 4. Results and Analysis

Figure 2 shows the distribution of five IGS stations used to estimate satellite clock errors, as well as station 'USNA' whose (static) coordinates are to be determined using the absolute positioning algorithm. Naturally, for this test, the coordinates of USNA are already known and are used to quantify the errors of estimation.

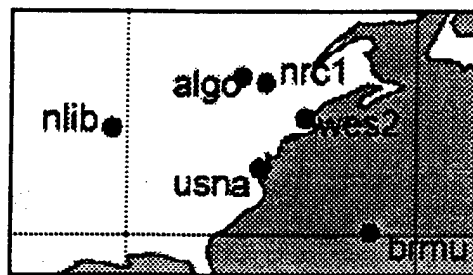


Figure 2

Table 1 shows the distances from the five IGS stations to station USNA.

Table 1: Baseline Lengths

From	To	Length (km)
NLIB	USNA	1214
ALGO	USNA	785
NRC1	USNA	722
WES2	USNA	582
BRMU	USNA	1291

To generate an orbital ephemeris for each GPS satellite with 30 sec resolution, a 9th-order Lagrange interpolator is applied to the corresponding IGS precise 900-second orbit. Figure 3

represents the difference between the interpolated X, Y and Z coordinates of the GPS satellite (PRN07) and the JPL 30-second orbits (JPL, 1999). Each component of the coordinates varies with standard deviation less than 3 cm. The mean differences are on the order of 1 cm or less. From the given standard deviations, we find the standard deviation of range difference to be 3.89 cm, which directly affects the satellite clock error estimation, as seen in (11).

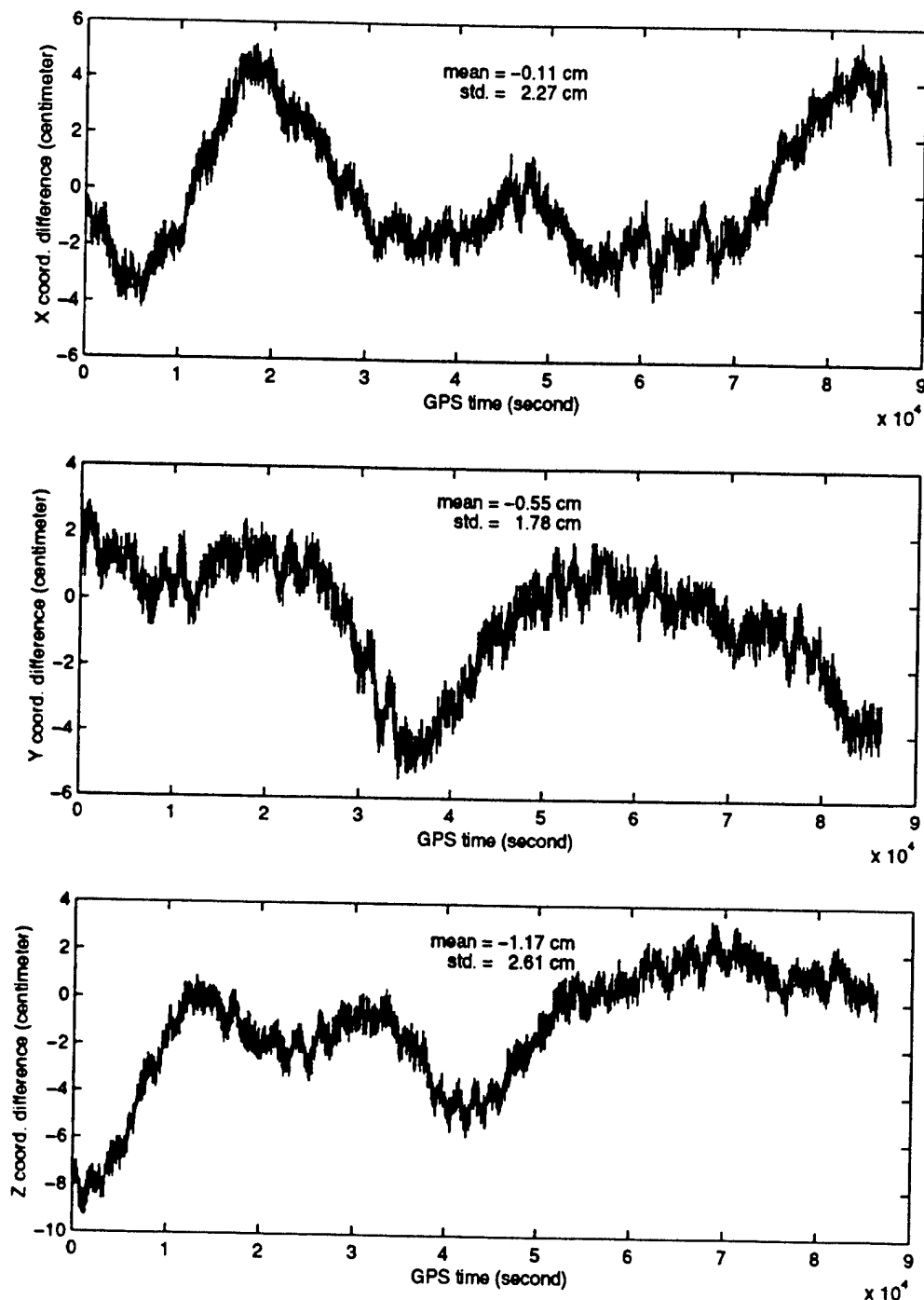


Figure 3: Differences between the interpolated IGS orbit and the JPL 30-sec. orbit for PRN07

Figure 4 shows the periodic general relativity effect (see Appendix) with respect to time. Using data from the navigation message provided at 2-hour intervals, 30-second values are calculated by the Lagrange 9th-order interpolator. In case of the PRN04 & PRN07 satellites, the effect increases up to 10 m, while for PRN01 & PRN29 the effect is much smaller, up to 1.5 m. In both cases, this effect is relatively large and the phase or pseudorange measurements should be corrected accurately for precise positioning and clock estimation.

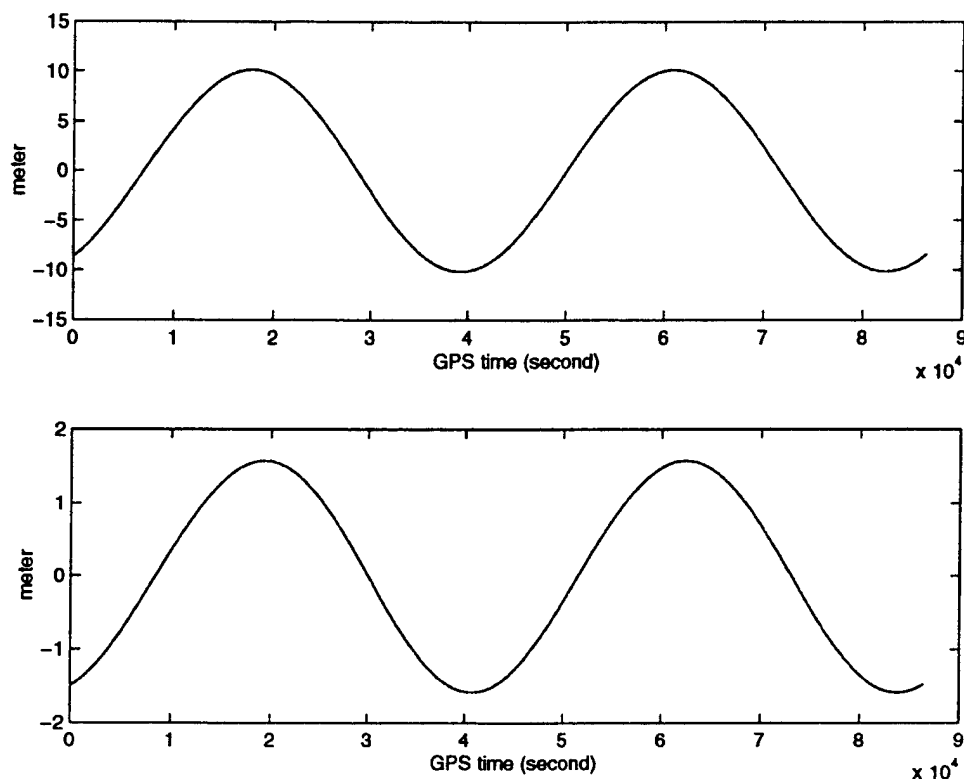


Figure 4: Periodic general relativity effect on satellite-differenced range; PRN04 & PRN07 (top), PRN01 & PRN29 (bottom).

Figure 5 shows the time- and satellite-differenced GPS clock error estimates,  $dt^{k,l}(t_{i,i-1})$ , for the PRN04 & PRN07 satellites, and their differences with respect to the corresponding estimates from JPL. Analogous results are shown in Figure 6 for satellites PRN01 & PRN29. To estimate these clock error differences, observations from five IGS stations are processed independently according to equation (11); and these redundant estimates are then averaged with equal weights. Figures 5 and 6 (top portions) indicates that the satellite clock error varies up to  $\pm 30$  meters at the 30-second resolution, most likely due to the Selective Availability. The standard deviations of the estimates with respect to the JPL estimates are less than 1 cm, with mean differences of about 0.1 cm.

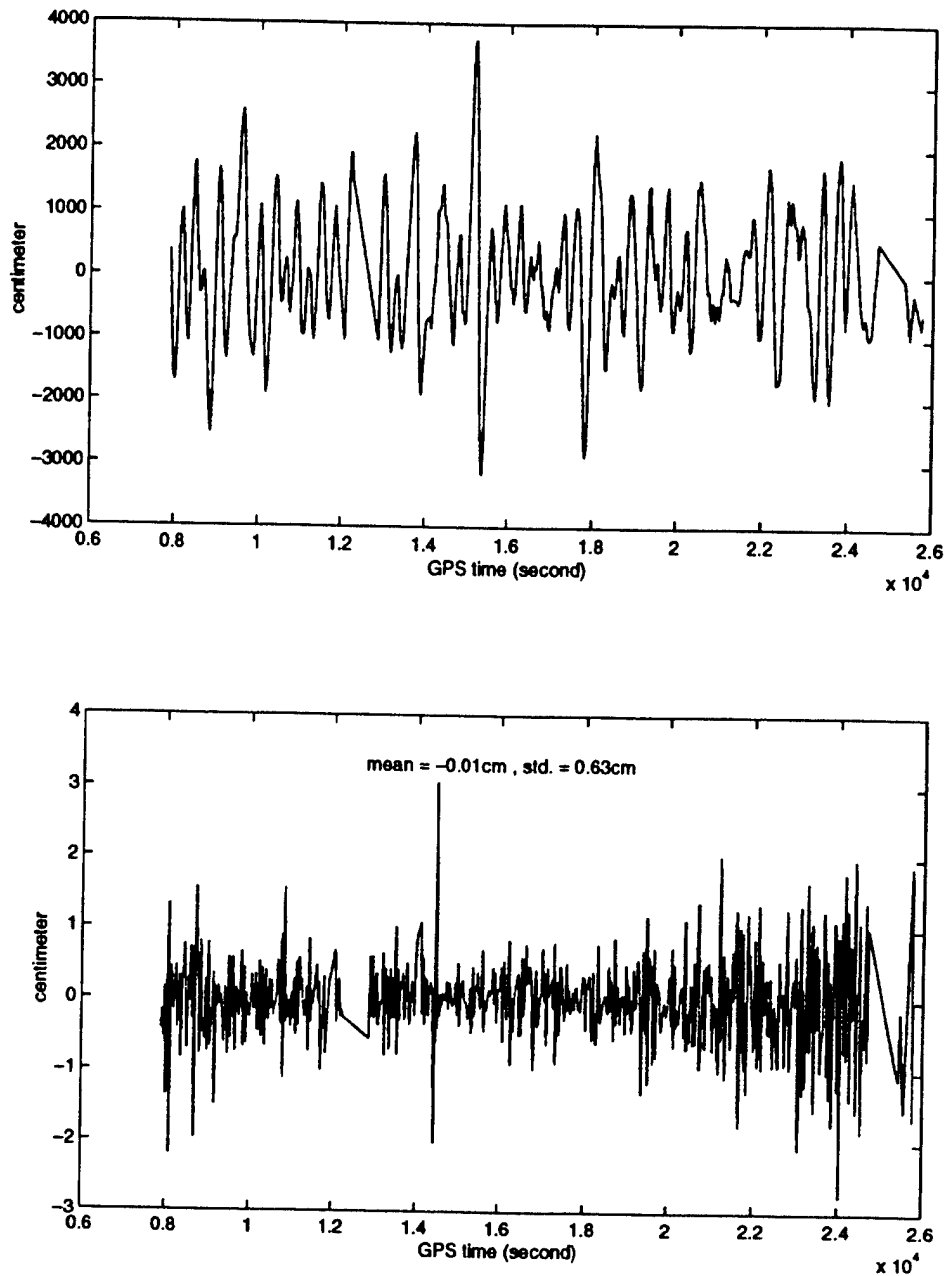


Figure 5: Time- and satellite-differenced GPS clock error (top) and differenced with JPL estimates (bottom) for PRN04 & PRN07.

The satellite-differenced GPS clock error estimates,  $dt^{k,l}(t_i)$ , and their differences with corresponding JPL estimates are shown in Figure 7 for PRN04 & PRN07 and in Figure 8 for PRN01 & PRN29. The estimates are calculated by summing the time-differenced estimates according to equation (14), and using the initial clock error estimate obtained from IGS. Again, these results are averages of five independent determinations using the five IGS stations. One can see the linear trends in the estimates as well as high frequency fluctuations for both pairs of



satellites. These linear trends could also be well determined using the navigation message's clock error information. The fluctuations in the error, however, cannot be corrected using the navigation message; and the magnitude of the fluctuations is about 200 nanoseconds (60 meters). Therefore, one cannot avoid an error of  $\pm 60$  m when performing a clock correction using just the navigation message.

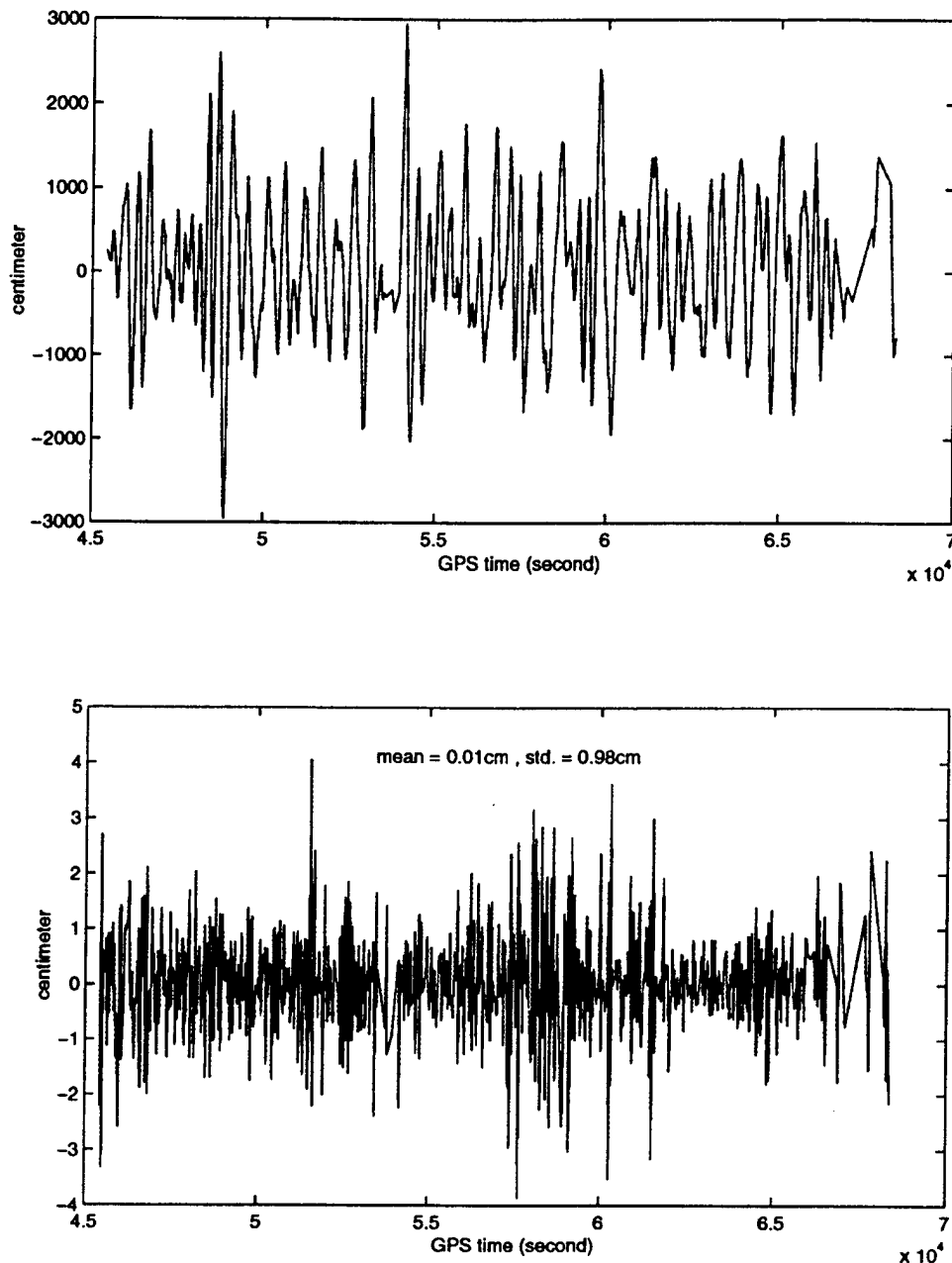


Figure 6: Time- and satellite-differenced GPS clock error (top) and differenced with JPL estimates (bottom) for PRN01 & PRN29.

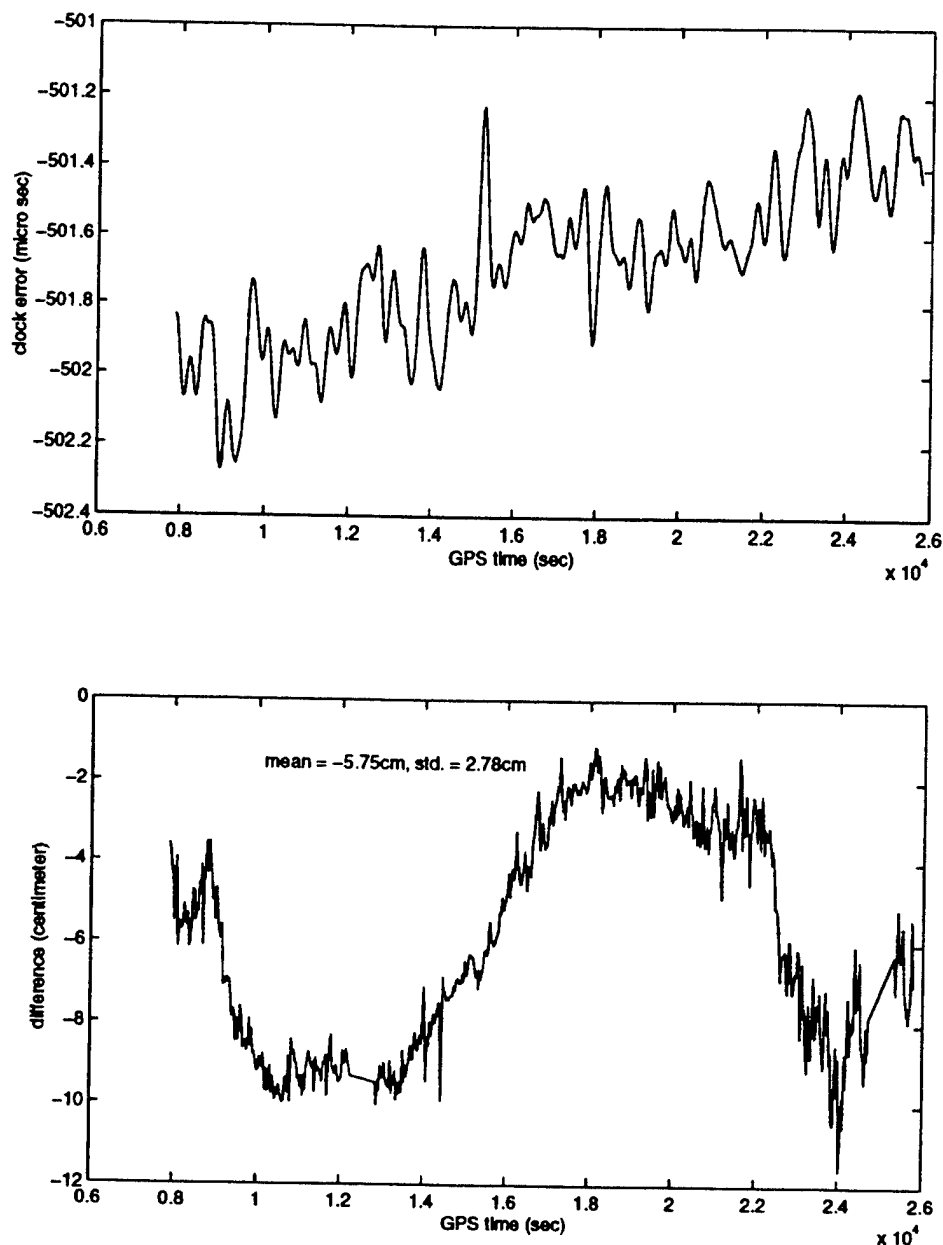


Figure 7: Satellite-differenced GPS clock error (top) and differenced with JPL estimates (bottom) for PRN04 & PRN07.

The differences between the clock error estimates Figures 7 and 8 (top) and JPL's estimates have standard deviations less than 4 cm, but still contain an offset and other significant systematic components, as shown in the corresponding bottom portions of these figures. It is not clear at this time what these are due to; however, some differences in the methods employed by JPL to process the data may explain them. In particular, we use:

- 1) Different orbits – Orbits affect the range determination directly, resulting in a difference in the

clock error estimation.

2) Modeled tropospheric delay – In our method, the tropospheric delay is not estimated but is modeled, while JPL estimates the zenith tropospheric delays and uses them to estimate the clock error.

3) First-order ionospheric effect – The ion-free phase combination used here eliminates only the first-order ionospheric effect, so the higher-order ion effects remain in the error.

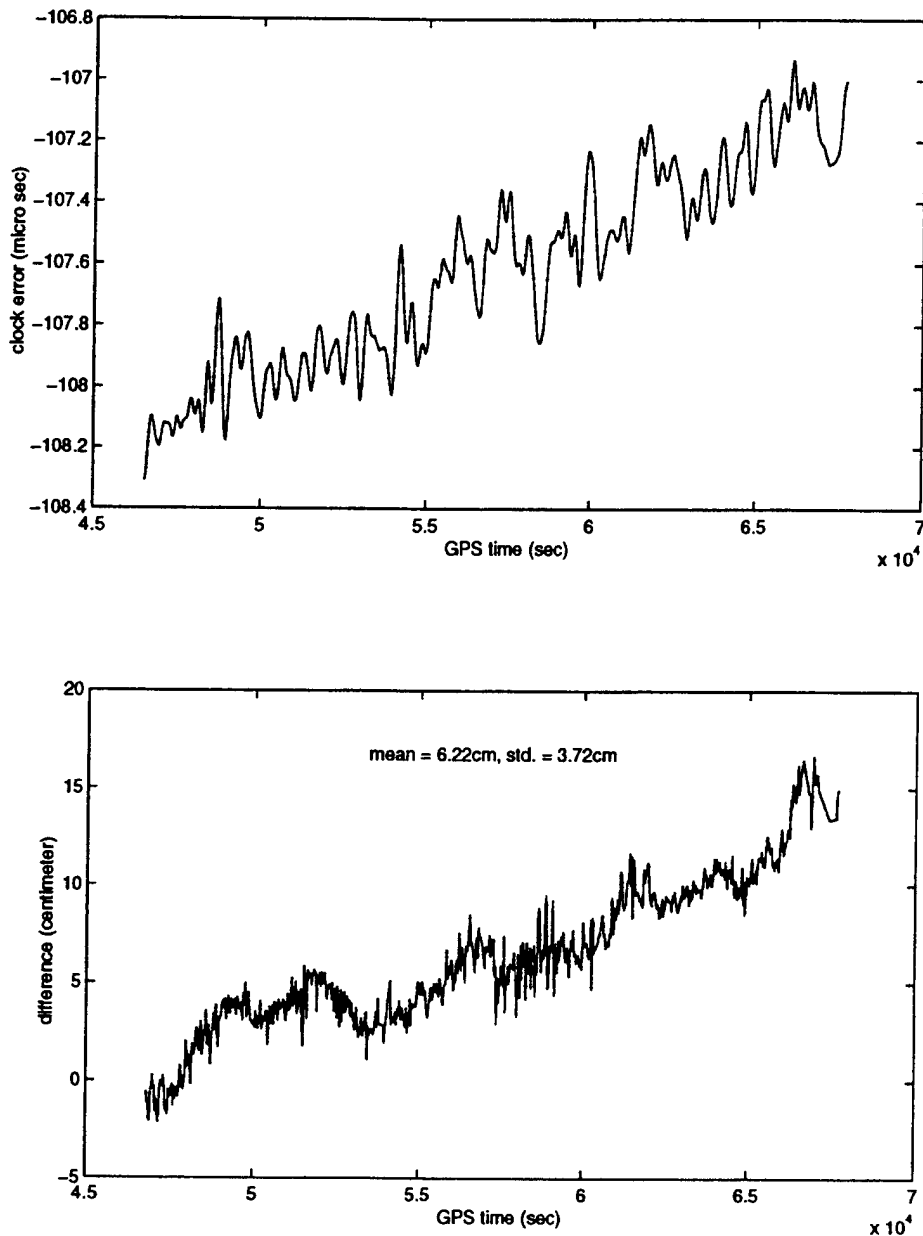


Figure 8: Satellite-differenced GPS clock error (top) and differenced with JPL estimates (bottom) for PRN01 & PRN29.

Using the GPS clock errors thus estimated (using known IGS stations and IGS orbits), precise absolute positioning is possible according to equation (24). One of the IGS stations is selected to check the algorithm in the case of a static receiver (further experiments are planned to test the algorithm for a moving platform). The station USNA is located at various distances (from 600 km to 1300 km) from the five IGS stations used to estimate the GPS clock errors (see Figure 2 and Table 1). Its coordinates are known and the clock error estimates determined from the other stations are used to estimate the absolute position of this station. This was done in a two-step process, whereby the ambiguities,  $\tilde{N}_w^{*k,l}$ , were first determined for the initial time using a standard ambiguity search algorithm (Leick, 1995). With these fixed, the absolute position for subsequent times was estimated on the basis of the single difference equation (24).

Figure 9 shows the differences between the known (Cartesian) coordinates and the estimated coordinates. The standard deviations are less than 2 cm for a span of 40 minutes of data; for longer periods the standard deviations would increase as more systematic effects enter (as also shown in the lower parts of Figures 7 and 8). In addition, there are relatively large mean differences up to 18 cm. These may be the result of imperfect ambiguity determination. Using the known coordinates of the station, the ambiguities could be determined much more accurately. In this case, the mean differences of the absolute position errors ranged from 3.4 cm to 6.3 cm.

## 5. Summary and Expectations

This report has shown the feasibility of determining very accurate coordinates of points in an absolute sense (point positioning) from GPS, if the GPS clock errors can be estimated. Algorithms for the estimation of these clock errors and for the determination of absolute position coordinates were developed and tested in the static mode. That is, they were tested at a point where the coordinates are always known (because it is stationary and its coordinates are well determined in the IGS frame). The results of this work showed that the algorithms thus developed are valid and absolute positions can be determined with a precision of 1 cm to 2 cm (continually up to 40 minutes) in all three coordinates. Biases on the order of 18 cm were also detected in the results, due probably to poor initial ambiguity determination.

These feasibility results must now be extended to kinematic applications. We plan to conduct kinematic tests using differential GPS kinematic position determinations as truth data. The main difficulty in this case is to get good estimates of the ambiguities. This will require further algorithm development and testing.

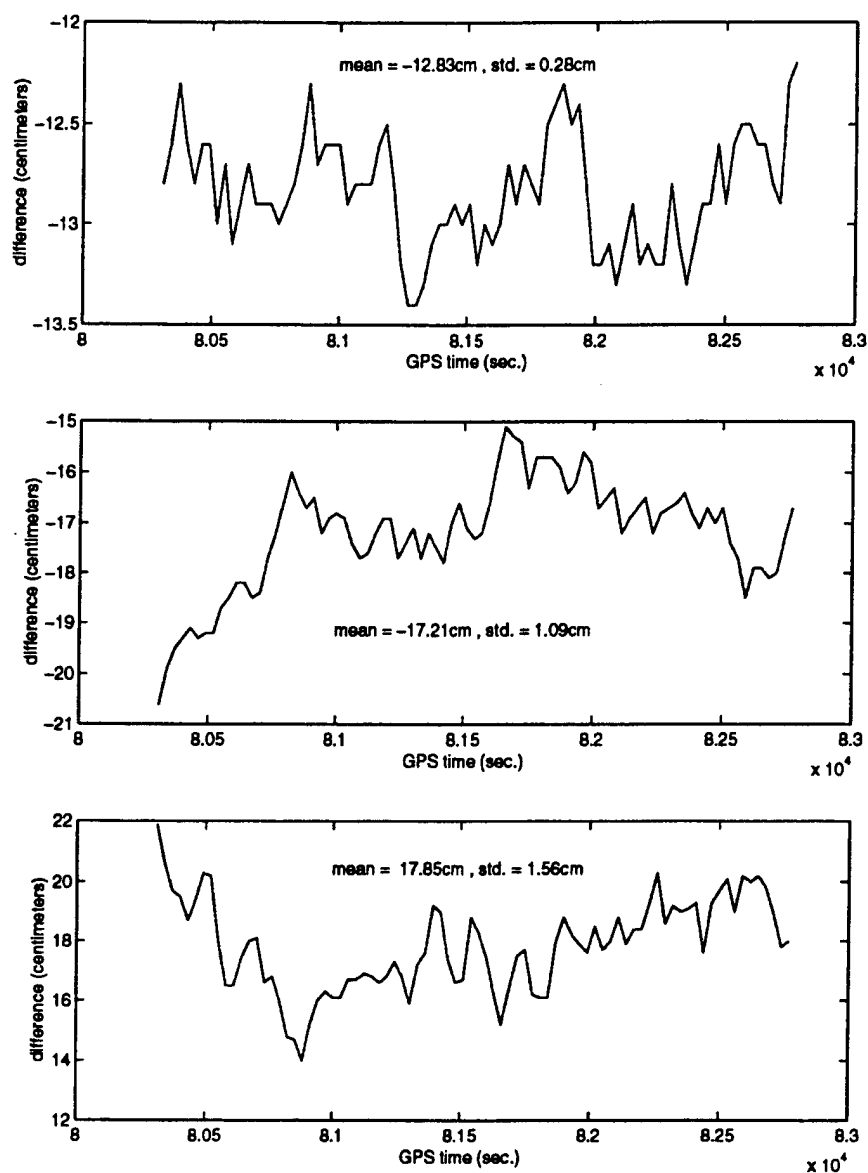


Figure 9: Differences between estimated coordinates of USNA and its known coordinates: x (top), y (middle), z (bottom).

**Acknowledgments:** This work was supported by a contract with U.S. Air Force Phillips Laboratory, no. F19268-96-C-0169, under sponsorship by the National Imagery and Mapping Agency (NIMA). The assistance and research of Mr. Jay Kwon, graduate student in the Department of Civil and Environmental Engineering and Geodetic Science, Ohio State University, contributed much to the success of this work.

## References

- Coco, R. (1991): GPS-satellites of opportunity for ionospheric monitoring. *GPS World*, 2(9), 47-50.
- Gibson, R. (1983): A derivation of relativistic effects in satellite tracking. Technical report, TR 83-55, Naval Surface Weapons Center, Dahlgren, Virginia.
- Goad, C.C. and L. Goodman (1974): A modified Hopfield tropospheric refraction correction model. Presented at the Fall Meeting of the American Geophysical Union, San Francisco, December, 1974.
- Goad, C. C. and Yang, M. (1995): A New Approach to Precision Airborne GPS Positioning for Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, 63(9), 1067-1077.
- IGS (1999): <ftp://cddis.gsfc.nasa.gov/pub/gps/products>.
- JPL (1999): <ftp://sideshow.jpl.nasa.gov/pub/jpligsac/hirate>.
- Hofmann-Wellenhof, B., Lichtenegger, H., and Collins, J. (1992): *GPS Theory and Practice* (4th revised edition). Springer, New York.
- Leick, A. (1995): *GPS satellite surveying* (2nd edition). Wiley-Interscience, New York.
- Remondi B.W., (1989): Extending the National Geodetic Survey standard GPS orbit formats. National Information Center, Rockville, Maryland, NOAA Technical Report NOS 133, NGS 46.
- Remondi B.W., (1991): NGS second generation ASCII and orbit formats and associated interpolation studies. Paper presented at the XX General Assembly of the IUGG at Vienna, Austria, August 11-24.
- Seeber, G. (1993): *Satellite Geodesy*. Walter de Gruyter, Berlin.
- Wilson, B.D. and Mannucci, A.J. (1993): Instrumental biases in ionospheric measurements derived from GPS data. JPL technical report, [www.jpl.nasa.gov/techreport/1993/1993-abs.html](http://www.jpl.nasa.gov/techreport/1993/1993-abs.html).

## Appendix - Periodic General Relativity Effect Correction

The general relativity effect, which is caused by 1) the difference between the gravitational field at the satellite and at the observing site, and 2) the motion of the satellite, is not considered because this effect was corrected in the factory before GPS satellite launch. Another effect arises due to the assumption of a circular orbit. The correction to the range measurement is given by Gibson (1983).

$$\delta_{rel} = \frac{2}{c} \sqrt{GM_E} a e \sin E$$

where  $e$  is the eccentricity of the orbital plane,  $a$  is the orbital semi major axis, and  $E$  is the eccentric anomaly of the satellite.